

An Evaluation of Natural Frequency, Two-body Loss Rate Constant and Visibility of Ramsy Fringes as a Function of Magnetic Field Strength

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(Received on : July 6, 2012)

ABSTRACT

Using the theoretical formalism of K. Goral *et al.*⁹, we have evaluated the natural frequency, two-body loss rate constant K_2 and visibility of Ramsy fringes as a function of magnetic field strength B_{evolve} (G). Our theoretically evaluated results are in satisfactorily agreement with the experimental data.

Keywords: Ramsy fringes, two-body loss rate constant, visibility of Ramsy fringes, Feshbach resonance, hyperfine states.

1. INTRODUCTION

The study of ultra cold molecules produced in atomic Bose and Fermi gases is one of the most important developments in cold atom physics¹. One of the most successful experimental techniques to produce molecules in atomic Bose-Einstein condensates utilized adiabatic sweeps of a magnetic field tunable Feshbach resonances level across the zero energy threshold of the colliding atoms. In this period a highly excited diatomic vibration bound states can

be efficiently produced²⁻⁶. There is a coherent superposition of atomic condensates and molecules. These experiments employed a sequence of two magnetic field pulses each of which rapidly approached the position of zero-energy resonance (the magnetic field strength at which the scattering length has singularity) on the side supporting the highly excited diatomic vibration bound state. The two pulses were separated by an evolution period of variable duration as a function of which the oscillations in the final condensate populations were observed. This achievement

offers the possibility of precise informatics measurements of the energies of the relevant states through the measurement of bulk properties of the gas.

In recent experiments^{7,8} the frequency and visibility of atom-molecule Ramsey fringes were observed. In this experiments, a Bose-Einstein condensate of ⁸⁵Rb atom in (F=2, m_F = -2) hyperfine states was exposed to a sequence of spatially homogeneous magnetic field pulses on the high-field side of the 155G Feshbach resonances. The evolution period of constant magnetic field strength B_{evolve} separated the pulse by an amount of time t_{evolve}. In the course of the experiments^{7,8} the final densities of atoms were recorded after expanding the cloud. The measurements were repeated with variable evolution time and magnetic field strength B_{evolve}.

Interference between different components of a partially condensed Bose gas was identified. There is a two different components of the final atomic cloud : a remnant Bose-Einstein condensate and a brest of atoms with a comparatively high mean kinetic energy. The magnitude of the brest of atoms and remnant condensate fractions exhibited an oscillatory dependence of the evolution time of the magnetic field pulse sequence. The existence of the third component of missing atoms was inferred from the difference between the initial and total final members of the detectable atoms. It was suggested that the missing atoms indicated molecular production and the oscillations were interpreted in terms of interference between atoms and molecules during the evolution period of the pulse sequence.

In this paper, using the theoretical formalism of K. Goral *etal.*⁹ we have theoretically evaluated the natural frequency ν_0 of Ramsey fringes, two-body loss rate constant K₂ and the visibility of Ramsey fringes as a function of magnetic field strength B_{evolve}. Our theoretically evaluated results are in good agreement with the experimental data^{7,8}.

2. MATHEMATICAL FORMULAE USED IN THE EVALUATION

One first considers a pair of ⁸⁵Rb atoms in a spherically symmetric trap exposed to a spatially homogeneous magnetic field^{10,11}. The degrees of freedom of the center of mass and relative motions of the atom pair are then exactly decoupled. The dynamics of the relative coordinate **r** is determined by the Hamiltonian of the form

$$H_{2B} = \frac{-\hbar^2}{2m} \nabla^2 + V_{\text{trap}}(r) + V(B, r) \quad (1)$$

Here V_{trap}(r) is the potential of the atom trap, m is the atomic mass and V(B, r) is the magnetic field dependent binary potential. In general, H_{2B} depends on all hyperfine states of the atoms which are strongly coupled by the Feshbach resonance state Φ_{res}(r). The analogy of the condensate mode is realized by the lowest energetic state of two atoms above the dissociation threshold of the pair interaction potential V(B, r) whose wave function Φ₀(r) obeys the stationary Schrodinger equation

$$H_{2B} \Phi_0(r) = E_0 \Phi_0(r) \quad (2)$$

The two-body Hamiltonian evaluated at the initial magnetic field strength of the pulse

sequence. The amplitude for the probability to detect the atoms in the state Φ_0 at the final time t_{final} of the pulse sequence provides the analog of the amplitude for the remnant condensate fraction and is determined by

$$T_{2B}(\Phi_0 \leftrightarrow \Phi_0) = \langle \Phi_0 | U_{2B}(t_{\text{final}}, t_0) | \Phi_0 \rangle \quad (3)$$

Here $U_{2B}(t_{\text{final}}, t_0)$ is the time evolution operator which satisfies the Schrodinger equation

$$i\hbar \frac{\partial}{\partial t} U_{2B}(t, t_0) = H_{2B}(t) U_{2B}(t, t_0) \quad (4)$$

$U_{2B}(t_{\text{final}}, t_0)$ can be factorized in the evolution operators $U_{no.1}(t_1, t_0)$, $U_{no.2}(t_{\text{final}}, t_2)$ and $U_{\text{evolve}}(t_{\text{evolve}})$ associated with the first and second magnetic field pulses and with the evolution period of the sequence respectively. Here t_1 is the time immediately after the first magnetic pulse t_2 is the time at the beginning of the second magnetic field pulse and $(t_2 - t_1) = t_{\text{evolve}}$ is the duration of the evolution period. The factorization of $U_{2B}(t_{\text{final}}, t_0)$ yields

$$U_{2B}(t_{\text{final}}, t_0) = U_{no.2}(t_{\text{final}}, t_0) U_{\text{evolve}}(t_{\text{evolve}}) U_{no.1}(t_{\text{final}}, t_0) \quad (5)$$

The frequency of the Ramsy fringe is obtained by equation

$$N_c^{\text{evolve}} = N_{\text{avg}} - \alpha t_{\text{evolve}} + A \exp(-\beta t_{\text{evolve}}) \sin(\omega_e t_{\text{evolve}} + \Delta\phi) \quad (6)$$

Here ω_e is the angular frequency

$$\omega_e = 2\pi(\nu_0^2 - (\frac{\beta}{2\pi})^2)^{\frac{1}{2}}$$

A is the amplitude of the interference fringes.

N_{avg} is the average number of atoms determined from the fringe visibility

α and β are fitting parameters

α = loss rate of atoms

β = damping rate

$\Delta\phi$ = phase shift of the Ramsy fringes

The two-body loss rate constant K_2 is determined by the formulae

$$K_2 = \frac{2\alpha}{N_{\text{avg}}} \langle n_c(t) \rangle \quad (7)$$

Where $\langle n_c(t) \rangle$ is the average condensate density.

The visibility of the Ramsy fringes are determined by the following equation.

$$V_{\text{Ramsy}} = \frac{1 - \exp(-4n\nu[P_{o,b}^{no.1}P_{b,o}^{no.2}])}{1 + \exp(-4n\nu[P_{o,b}^{no.1}P_{b,o}^{no.2}]^{\frac{1}{2}})} \quad (8)$$

Here $P_{o,b}^{no.1}$ is the probability of molecular production in the first magnetic field pulse and probability is $P_{b,o}^{no.2}$ their reconversion into atom pairs in the lowest energetic quasi continuum state Φ_0 (r). n is the homogeneous gas density and ν is the volume.

3. DISCUSSION OF RESULTS

In this paper, we have determined the natural frequency ν_0 , two-body loss rate constant K_2 and visibility of Ramsy fringes all as a function of magnetic field strength B_{evolve} (G). The theoretical results were

compared with the experimental data^{7,8}. The evaluation has been performed with the help of theoretical formalism developed by K. Groal *et al.*⁹. The results are shown in table T1, T2 and T3 respectively. In table T1, using equation(6) we have calculated the natural frequency ν_0 of Ramsy fringe. Our evaluated results are lower than the experimental data^{7,8} in magnitude but the trend is the same. The natural frequency ν_0 increases with magnetic field strength B_{evolve} (G). Our theoretical results also show that for $B_{\text{evolve}} > 158\text{G}$ which is far away from the zero –energy resonance at $B_0 = 155\text{G}$, the fringe frequency closely matches with the vibrational frequencies associated with weakly bound molecular state^{10,11}. The two-body loss rate constant K_2 is determined using equation (7) and results are shown in table T2 with the experimental data. The rate constant K_2 indicates the loss of condensate atoms due to energy transfer from the magnetic field pulses. This derives initially weakly interacting Bose-Einstein condensates into a strongly correlated non-equilibrium state. In this calculation the deeply inelastic spin relaxation phenomena has not been included¹². Our theoretically evaluated results of loss constant K_2 are in good agreement with the experimental data^{7,8}. Visibility of Ramsy fringes are determined using equation (8) and the results are shown in table T3 along with the experimental data^{7,8}. The evaluation has been performed using exact transition probability $P_{o.b}^{no.1}$ and $P_{b.o}^{no.2}$ for both the pulses of realistic sequence and their counterparts as a function of magnetic field strength¹³. Our theoretically evaluated results show that the fringe visibilities

decreases with the increase of magnetic field strength and for high field the decrease is almost constant. Our theoretically evaluated results are in satisfactorily agreement with the experimental data.

Table T₁
An evaluated results of the natural frequency ν_0 of Ramsy fringes as a function of magnetic field strength B_{evolve} (G)

B_{evolve} (G)	Natural frequency ν_0 (MHz)	
	Theoretical results	Expt. ^{7,8}
155	2.85	4.22
156	4.32	5.36
157	8.87	9.49
158	22.36	24.55
159	46.54	50.21
160	73.29	80.56
161	116.10	120.49
162	176.56	184.26
163	219.22	224.16
164	284.27	320.59
165	328.56	357.15

Table T₂
An evaluated results of two –body loss rate constant K_2 as a function of magnetic field strength B_{evolve} (G)

B_{evolve} (G)	Two-body loss rate constant $K_2(\text{cm}^3/\text{s})$	
	Theoretical results	Expt. ^{7,8}
156	2×10^{-8}	3.16×10^{-8}
156.5	3.14×10^{-8}	3.14×10^{-8}
157	3.05×10^{-8}	1.97×10^{-8}
157.5	2.84×10^{-8}	1.62×10^{-8}
158	2.56×10^{-8}	1.48×10^{-8}
158.5	2.15×10^{-8}	1.29×10^{-8}
159	1.87×10^{-8}	1.17×10^{-8}
159.5	1.62×10^{-8}	1.05×10^{-8}
160	1.47×10^{-8}	0.92×10^{-8}
165	1.16×10^{-8}	0.86×10^{-8}

Table T₃
An evaluated results of visibility of Ramsy
fringes as a function of magnetic field
strength B_{evolve} (G)

B _{evolve} (G)	Fringe visibility	
	Theoretical results	Expt. ^{7,8}
156	1.052	0.987
157	0.924	0.895
158	0.814	0.795
159	0.692	0.714
160	0.634	0.625
161	0.548	0.524
162	0.509	0.493
165	0.488	0.425
170	0.423	0.407

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